Response Time Considerations in Temperature Sensing

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Abstract

The response time of a temperature sensor is an important performance parameter that is often misunderstood, or at best, poorly understood. Response time is a measure of how quickly a sensor responds to changes in temperature and is affected by the size of the sensor, the internal construction of the sensor and the environment itself. Each particular application will define how fast the sensor needs to respond to changes in temperature — the fastest response time may not be necessary or practical. The consistency of response time across multiple sensors in an application is often more important than a specific response time value. This will ensure consistent performance across multiple installations. This technical treatise includes theoretical calculations but is balanced with visual explanations to provide a practical understanding of response time.

Response Time and Temperature Sensors

An important performance characteristic of temperature sensors is response time, which is a measure of how quickly a sensor can measure a change in temperature. Specifically, response time is described in terms of a time constant — which is the time necessary for a temperature sensor to respond to a 63.2% step change in temperature. This step change is most often tested by rapidly inserting a sensor at room temperature into a heated liquid (often water or oil). This method allows for a relatively consistent measurement that describes the speed of a temperature sensor's reaction to changes in its environment. Even though most temperature sensors do not experience an instantaneous step change in temperature during actual use, the time constant is useful for comparing the relative performance of different sensor types or analyzing variation within a production batch. To illustrate, imagine the temperature sensor in a home oven. The temperature in the oven does not instantly change temperature but instead heats over a period of time to the set point. While the step change definition of response time does not match many actual temperature sensor usage environments, the response time value is still a useful comparison tool as sensors with a faster step change will also respond faster to a temperature ramp and minimize temperature overshoot.

Theory

The time constant theory comes from the Lumped Capacitance Model, a simplified model of a thermal system (the equivalent in electronics is called the "Lumped Element Model").

The lumped capacitance model is used to describe thermal systems in which objects are warmed or cooled uniformly — such that the heat transfer from the object is proportional to the temperature difference between the object and the fluid it is immersed in. The time constant, **τ**, according to the lumped capacitance model, should be constant through the response curve. In the standard form of the equation, the time constant is equal to the time required for the object to undergo 63.2% of a step change in temperature.

LUMPED CAPACITANCE MODEL:

$$\frac{T_f - T}{T_f - T_0} = e^{\left(\frac{-t}{\tau}\right)}, \text{ where } \tau = \frac{\rho V c}{h A_s}$$

T = temperature (K)
T_0 = initial temperature (K)
T_f = fluid/final temperature (K)
t = time (s)
τ = time constant (s)
ρ = density ($kg \cdot m^{-3}$)
V = volume (m^3)
c = specific heat capacity ($J \cdot kg^{-1} \cdot K^{-1}$)
h = convection coefficient ($W \cdot m^{-2} \cdot K^{-1}$)
A_s = surface area (m^2)

Note: The equation can be modified for other time constant percentages (chiefly $\tau\%$ = 0.5 and 0.9):

$$\frac{T_f - T}{T_f - T_0} = \left(\frac{1}{1-\tau\%}\right)^{\left(\frac{-t}{\tau}\right)}$$

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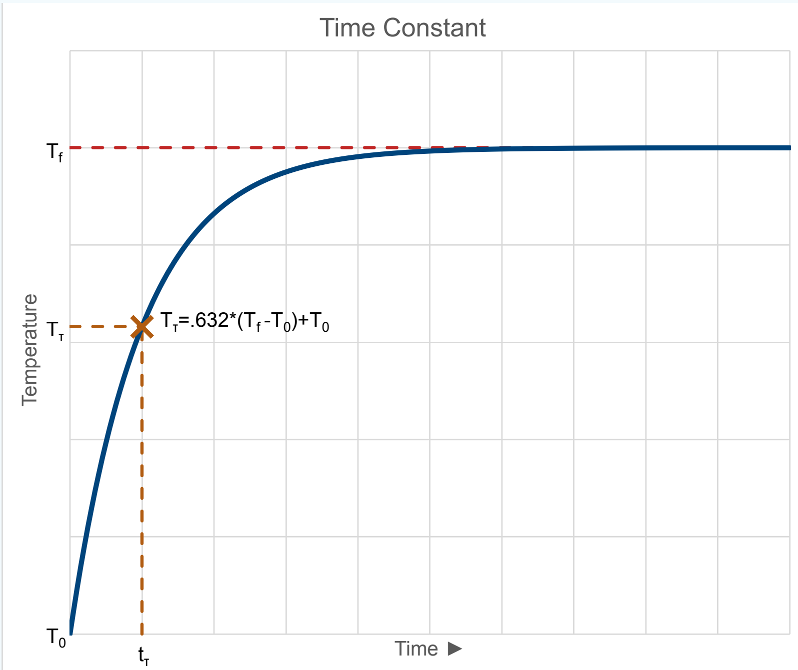

The lumped capacitance model is one of the easiest transient thermal heat transfer analyses to apply, but it is most useful for uniform solids. A key assumption of this model is that temperature difference across/within an object is much smaller than that between the object's surface and the fluid it is immersed in. For uniform solids, this means that the resistance to conduction within the object must be very low compared to the resistance to convection across the fluid boundary layer. This ratio is given by the Biot number, and can be used as a first estimate of the validity of the lumped capacitance model. It should be noted that in non-uniform solids, internal geometries can play a significant role in the transient temperature response. Therefore, as one increases the complexity of temperature sensor internal geometry, one would expect less adherence to the lumped capacitance model.

BIOT NUMBER:

$$Bi = \frac{R_{cond}}{R_{conv}} = \frac{\text{Resistance to conduction within "solid"}}{\text{Resistance to convection across fluid boundard layer}} = \frac{hL}{k}$$

R = thermal resistance $(K \cdot W^{-1})$
h = convection coefficient (K)
L = characteristic length (m), $[L \equiv V/A_s \Rightarrow R/2$ *for a long cylinder]*
k = thermal conductivity $(W \cdot m^{-1} \cdot K^{-1})$

Calculation of the Biot number will reveal that:

$Bi \leq 0.1$	Lumped capacitance model is valid (it models transient response well, small error). High internal conductivity compared to heat transfer across boundary layer; low temperature gradients within.
$Bi \approx 1$	Lumped capacitance model may not be valid. Comparable conduction/convection. Method may still be acceptable for certain internal geometries.
$Bi >> 1$	Potentially large error from using lumped capacitance model. Significant temperature gradients within solid, due to low thermal conductivity or internal geometry.

Below is a rough calculation of the Biot number for two types of temperature sensor probes. WW is a lightweight, wire-wound sensing element that is in close contact with the inside wall of the stainless steel protection tube and is designed for fast response time. Alternatively, TF is a probe with a thin-film element where the element is not in close contact with the protection tube as there are additional materials (epoxies and insulating tubing) between the sensing element and the wall of the protection tube. In most applications, either sensor construction is perfectly acceptable, although one or the other of these constructions may be better for certain situations.

Moving Water (1m/s)

$Bi_{WW} =$	1.71	✕
$Bi_{TF} =$	35.2	✕

Still Oil

$Bi_{WW} =$	0.0128	✓
$Bi_{TF} =$	0.263	✕

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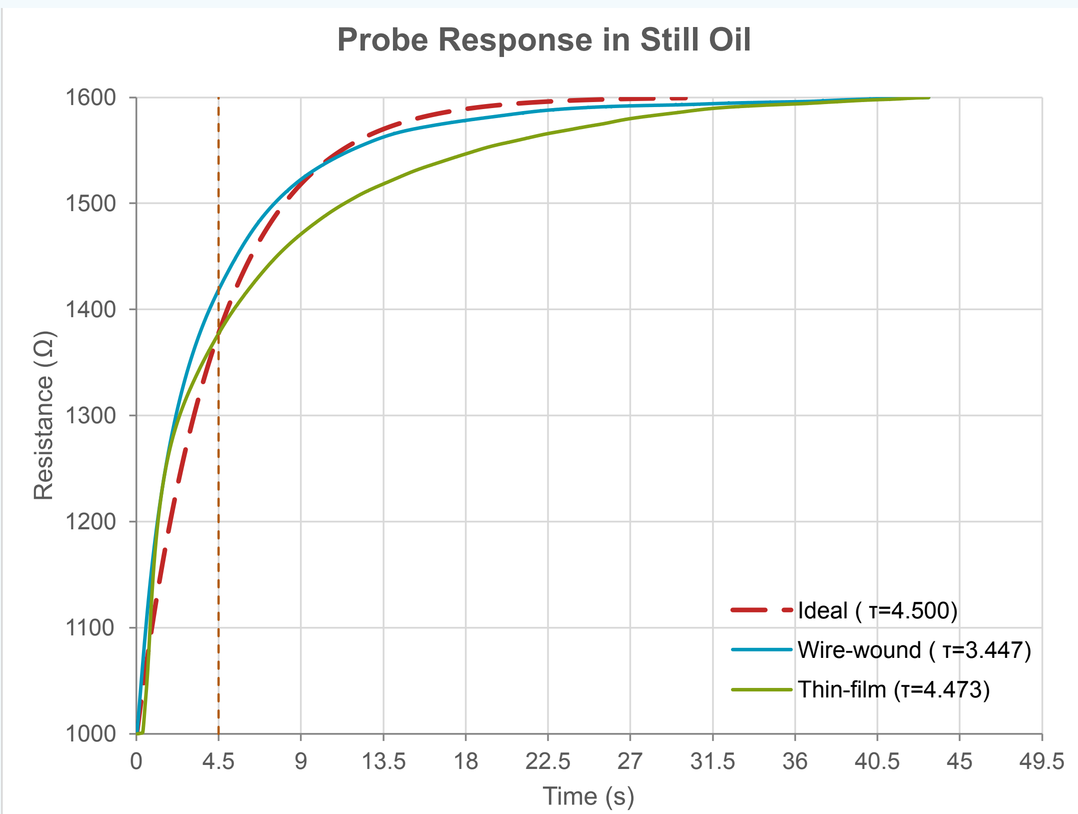
Practical Considerations

The most important consideration from the time constant theory (concerning time constants, anyway) is that the farther the part geometry deviates from a simple lump of uniform material (and the more thermally resistive it becomes), the less constant the “time constant” is going to be. The fastest 63.2% step change will occur the instant the probe hits the fluid. As time increases, the response curve will deviate from the exponential “ideal” curve, and each 63.2% step change will be longer.

The graph below depicts testing on the temperature probes with the Biot number calculations above. An ideal curve is plotted in red (dashed line), and the 63.2% time constant of 4.5s for this curve is shown as a vertical line. Both parts have a faster first time constant than the ideal curve... yet the ideal approaches the fluid temperature more quickly than either of the parts measured. As expected (via Biot calculation above), the wire-wound parts are closer to the ideal curve.



The gradual increase of time constant over the course of a test is an important consideration. It reveals that in order to obtain consistent results, it is imperative (at least as a first consideration) that measurements are taken as early in the response curve as possible, so as to capture the first 63.2% step change.

Still Fluids

Capturing the first 63.2% of a step change can be challenging, especially in still fluids. In the case of still fluids, the only agitation occurs by the natural movement of the fluid due to buoyancy forces. By the mere act of inserting a temperature sensor into a still fluid, there is a period of forced convection while the sensor is moved into position and the fluid is agitated by the sensor. Therefore, experimentally measuring a perfect first step change in temperature is essentially impossible in a still fluid.

The still fluid issue is most pronounced in fluids with high heat transfer coefficients (e.g. not air). In fluids with low heat transfer coefficients, the insertion time is much lower in proportion to the first time constant, so less error is contributed to the value.

It is important, when measuring the time constant of sensors in still fluids, to do one or more of the following:

- Insert the part as quickly as possible (reduce the proportion of insertion time to time constant).
- Insert the part consistently. Ideally, use a repeatable mechanical contraption.
- Begin data gathering at a consistent point (this will increase measurement consistency, but not necessarily accuracy).

Testing in Water

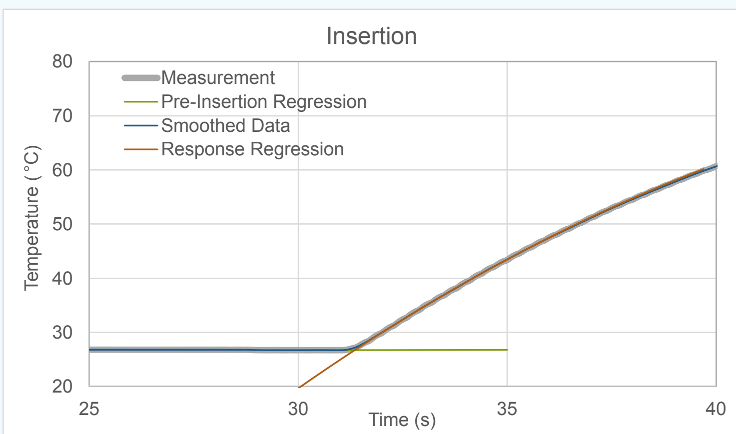
For the lumped capacitance model to be applicable, the resistance to conduction within the object must be very low compared to the resistance to convection across the fluid (thermal) boundary layer. The implication, then, is that water (which has a relatively high heat transfer coefficient) will be one of the worst fluids to test in and still expect adherence to the lumped capacitance model. That's unfortunate, being that it's ubiquitous! The other implication is that high thermal conductivity materials will yield better adherence to the lumped capacitance model (more constant time "constants"), if not necessarily the quickest response times (which are influenced by heat capacity).

Insertion

Observe the ideal curve plotted in any of the graphs above: The temperature (or resistance) response appears to climb at a steep slope at time=0. This is not reality. The first derivative of the temperature sensor response will always be continuous and smooth. Therefore, (unless the data is truncated) there will always be a “lead-in” to the response curve that occurs when the sensor is inserted into the test fluid.

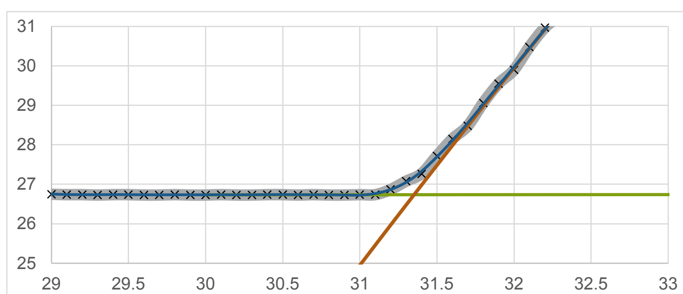
This lead-in is unavoidable, and it makes calculation of the time constant troublesome. It is a similar problem to still fluids, but different in that error comes primarily from the amount of part that is inserted into the fluid, not the fact that the convection heat transfer rate is being modified by the insertion process. The lead-in is most troublesome when it is relatively large in comparison to the time constant of the part. It can also be an issue when comparing time constants obtained on different measurement apparatuses or obtained by different people.

Most of the time, the delay contributed to the time constant by the insertion of the part is lost within the variability of data (repeatability of the test, part-part variability, etc.). If the insertion time is very small compared to the time constant, rigorously eliminating this portion of the data will not significantly alter the data spread and may not even alter the average to a degree that’s worth considering.



There are occasions, such as hand-dunking fast responding sensors, where consistent elimination of the insertion data would be useful. The charts on this page depict the response from a temperature sensor inserted into an air flow at 6 m/s. It can be seen from the second chart that the insertion period could contribute up to 0.25 seconds to the response time, depending upon which point is chosen as the start of measurement.

These parts were dropped into the air flow by gravity. If they had been inserted by hand, the “lead-in” part of the curve could vary much more than is depicted here, and the resulting variably makes detecting the true time of insertion via computer algorithm difficult. One method of consistently eliminating the insertion time from the time constant data is to eliminate the “lead-in” from the data. By drawing regression curves on the pre-insertion data and a small part of the response curve, one can find the intersection point that would mimic an ideal response.

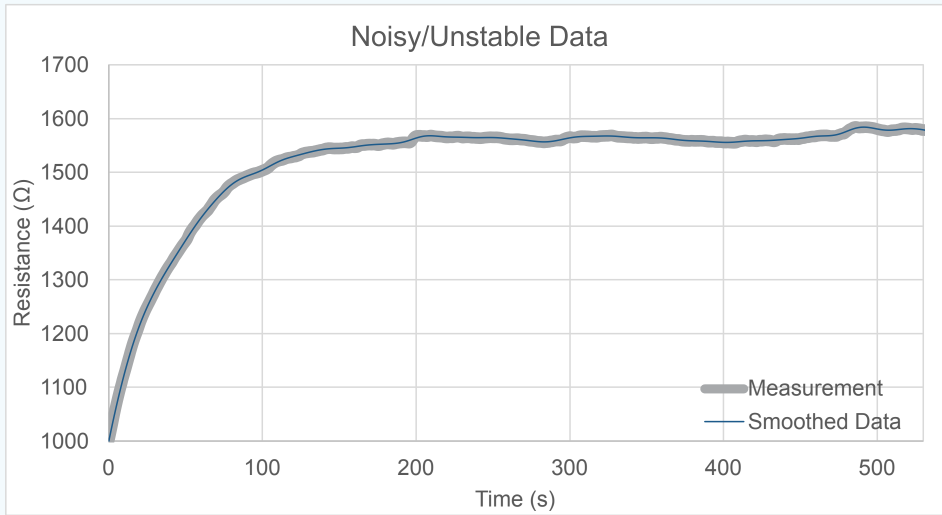


The second chart is an enlarged section of the above graph focused around the insertion time with the applicable regression curves depicted (with intersection at about 31.3s). In order to deliver consistent detection of the points around the insertion point, some smoothing/noise reduction (blue line) was applied to the measurement data. If applied to all parts in a set of measurements, the resulting point can be used as the actual insertion time (t=0).

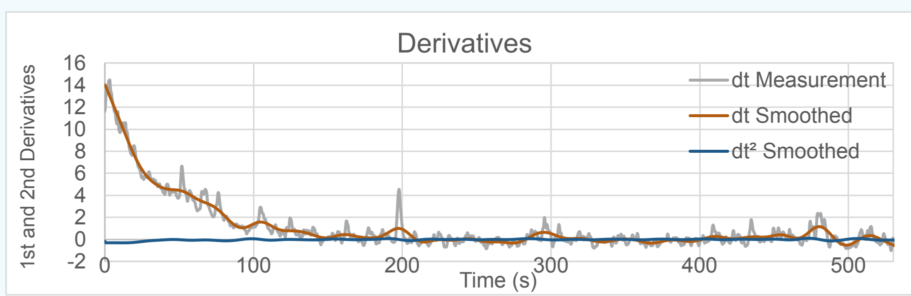
Noisy/Unstable Data

It is inevitable that data will contain some noise or instability. In the data below (taken from a probe in an essentially still-air test where the probe has been exposed to a red-hot heating element on one side), there is considerable noise in the section of data that should be stable.

Where does the stability start? What is the stable temperature (or resistance) value? Even smoothing the data doesn't appear to help much. Fortunately, smoothing the data does allow the first and second derivatives to be used to figure out where significant points in the data are. In the scenario below, if the data is traversed forward from t=0, one can find the point where the second derivative drops below a certain value (perhaps 0.5). A subset of the remaining data (such as the last 2/3 from this point) can then be averaged to determine the final temperature (or resistance).



Note: The data appears to be very noisy, but it's actually not that bad due to the length of time that the data was taken over (nearly 9 minutes). Data taken on quicker probes can look fairly smooth, but have nasty first derivatives due to cyclic noise in the rapid data capture.



Instantaneous Time Constant Calculation

So far, all the time constants I've discussed have been via what I'd call the "lookup" method: Find the point in the data where the 63.2% step change happens, then figure out how much time it took to get to that point. Each subsequent time "constant" can then be figured out the same way. The alternative is to calculate the instantaneous response time at all points using the lumped capacitance model. Rearranging the equation, one gets:

$$\tau = \frac{-t}{\left(\frac{T_f - T}{T_f - T_0}\right)}, \text{ or for decay: } \tau = \frac{-t}{\left(\frac{T - T_f}{T_f - T_0}\right)}$$

Calculating the time constant at any arbitrary point in the data isn't particularly helpful. There are several points where it does provide useful information:

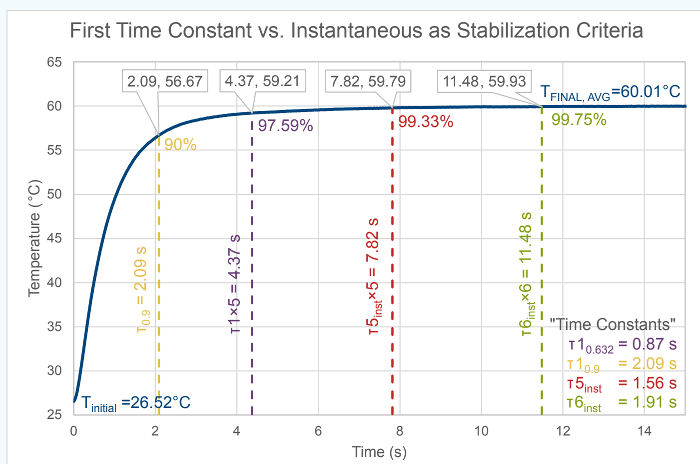
- Calculating each time constant (τ1=.632, τ2=.865, τ3=.950,τ4=.981, τ5=.993, τ6=.997). Unlike the "lookup" method, the value generated at each of these points will be the average of all "time constants" up to and including the point where it is calculated, (e.g. $\tau 3_{instantaneous}$ =average ($\tau 3_{lookup}$+ $\tau 2_{lookup}$= $\tau 1_{lookup}$))

- Reviewing the consistency of time constant. If constant, the part likely adheres well to the lumped capacitance model, and more/better assumptions can be made about time constants in other fluids/flow conditions.

The instantaneous time constant is relatively useful out to 6 time constants (whereas the "lookup" method can suffer from noisy data beyond 4 time constants), and may provide the best picture of how parts will actually perform. Since the instantaneous time constant (at the % value of a particular "lookup" time constant) is an average of all prior "lookup" time constants, it gives a more accurate picture of product performance. For example, the following figure is the response plot of a part that has been tested in 1 m/s moving water. The $\tau 1_{0.632}$×5 (the normal rule of thumb) is plotted along with $\tau 5_{inst}$×5 and $\tau 6_{inst}$×6. With good adherence to the lumped capacitance model, $\tau 1_{0.632}$×5 should yield a 99.33% change. Because the time constants are not constant, it does not! $\tau 5_{inst}$×5 yields the true 99.33% change, making the τ5inst value a better representative of time constant. $\tau 6_{inst}$ 6 is also plotted, and yields a true 99.75% change. Conclusion: $\tau 5_{inst}$ is a much better practical measure of time constant than the first time "constant". It doesn't look quite as good on paper, though.

For a part that adheres well to the lumped capacitance model, it is often stated that 5 time constants is roughly equivalent to a fully stabilized (final) value. The assumption, then, is that a published "time constant", $\tau_{0.632}$, can be multiplied by 5 to determine how long a part will take to stabilize. In my experience, most temperature sensors are too complex to adhere rigidly to the lumped capacitance model. For the most part, the time constant is not particularly constant, and will tend to increase as the sensor nears the stabilization.

Obviously, this issue can be remedied by selecting another stabilization criterion, such as 99% time constant instead of 63.2%. Such a criterion is not likely to be adopted by many manufacturers since at initial glance, it's a much higher value than everyone else publishes. 90% step-change (often abbreviated or τ0.9) time constants are published fairly frequently and are a better indicator of performance than the 63.2% version.

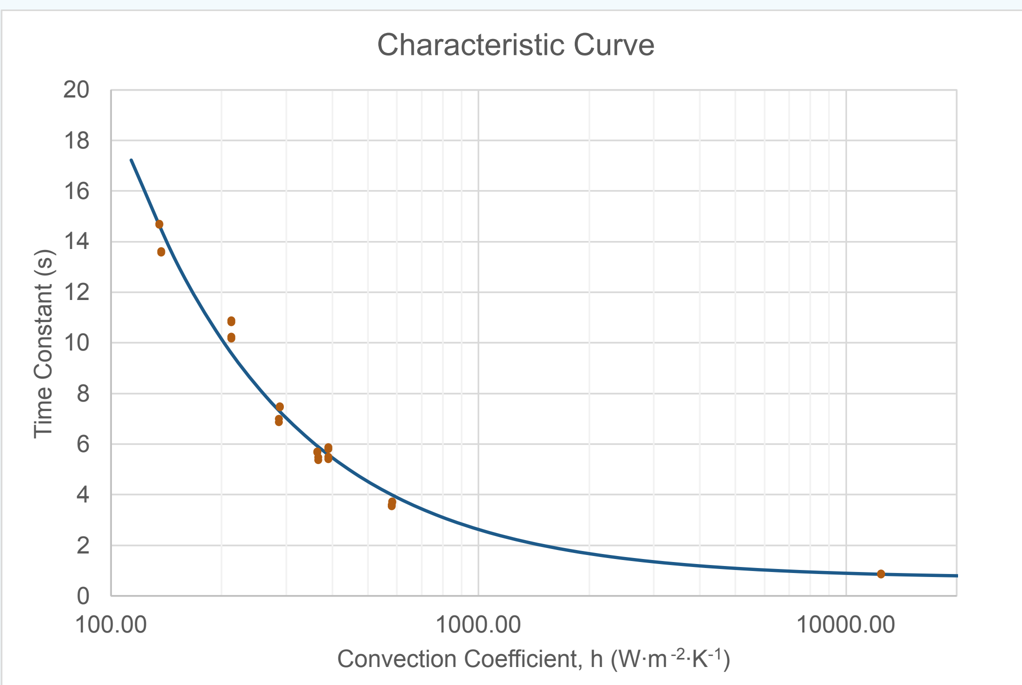


Output Temperature Linearity

Time constant is based upon temperature. For a rigorous approach, all sensor outputs should be converted to temperature before calculating the time constant. Practically, inputs that are linear or close to linear with temperature can be used. Most RTDs and thermocouples (with possible exception of type K) and ICs are linear enough to introduce little error over the typical response time temperature range (i.e. sub 100°C). Thermistor response should definitely be converted to temperature. Non-linear RTDs (Ni, NiFe) and TCs should probably be converted if large temperature changes (>100°C) are used for time constant testing.

Characteristic Curve

A characteristic curve can be generated that describes the time constant for any fluid flow condition. The time constant is a function of convection coefficient, h. Plotting the time constant of a part (or many parts) against the calculated convection coefficient of a particular test fluid allows one to generate a characteristic curve. Examining the lumped capacitance equation, one can see that $\tau \sim 1/h$. Using three or more time constant measurements, one can fit the data to a power function of the form $\tau(h) = ah^b + c + \overline{\tau}_0$ (where $\overline{\tau}_0$ is the average time constant at some arbitrary test condition, such as 1 m/s water, which the curve is forced to pass through during curve regression). The variable b can be allowed to vary for the purposes of obtaining a good fit, but it should remain very close to -1 (if it's not, something is probably wrong with the data). With sufficient data, a specific part or type of parts can be characterized, then ($\overline{\tau}_0$) τ and the coefficients can be used to calculate the time constant in any other fluid/flow condition.



Design for Low Time Constant

From the equation shown in the first section, it's apparent that the following can be done to design for a low time constant:

- Reduce density
- Reduce specific heat capacity
- Reduce volume
- Increase surface area

Specific heat capacity and density reduction are often the most effective options (the combo is effectively a reduction in volumetric heat capacity – which is not a property that is often published anywhere).

Increasing thermal conductivity will also have an effect on time constant, but the bang-for-your-buck is dependent upon external heat transfer rate. It's therefore not necessarily as effective as reducing specific heat capacity or density (which are effective for any external flow conditions). Increasing thermal conductivity will decrease the Biot number. Internal heat transfer rate will be increased relative to the external heat transfer rate. The effect of changes in thermal conductivity will therefore be most apparent for high external heat transfer rates.

Alternatively, there may be sensor applications where a slower response time is desired. This would most likely occur where a transient condition affects the temperature sensor but not the actual system being measured. An example of this could be a residential thermostat. If the thermostat was touched by a warm hand, you would not want the sensor to immediately indicate a temperature change and turn on your air conditioning. If slower response time was desired, the design recommendations above would be reversed.

Whether lower (most often) or higher response time is required, it is just as important that a given temperature sensor design has a consistent response time. This will ensure consistent and reliable performance across all sensors used in an application (each thermostat in this example should behave the same to achieve expected results).

Thermal conductivity is an attractive material property and may get more focus than it deserves from a design standpoint. To illustrate this point and help explain the effects of internal and external heat transfer on system, a simplified analogy is follows:

External heat transfer (convective):

- Low/slow: Trucks deliver 1 package every 10 minutes
- High/fast: Trucks deliver 1 package per minute to a building.

Internal heat transfer (conductive, or thermal conductivity):

- Low/slow: Conveyor belt takes 1 minute to deliver packages from trucks to operators.
- High/fast: Conveyor belt takes 0.5 minutes to deliver packages from trucks to operators.

In this analogy, a truck delivers packages to a building and inside the building a conveyor belt delivers packages to operators. The trucks are equivalent to the external (convective) heat transfer coefficient and the conveyor belt is equivalent to the internal heat transfer coefficient (thermal conductivity).

Case 1: Slow external system

- The factory opens for the day (time starts).
- Package 1 arrives 10 minutes later. It is placed on the conveyor.
- Package 1 takes 1 minute to ride the conveyor belt and is removed by an operator at minute 11.
- Package 2 arrives at minute 20. It is placed on the conveyor.
- Package 2 takes 1 minute to ride the conveyor belt and is removed by an operator at minute 21.
- Total time for 2 packages: 21 minutes

- The factory opens for the day (time starts).
- Package 1 arrives 10 minutes later. It is placed on the conveyor.
- Package 1 takes 0.5 minutes to ride the conveyor belt and is removed by an operator at minute 10.5.
- Package 2 arrives at minute 20. It is placed on the conveyor.
- Package 2 takes 0.5 minutes to ride the conveyor belt and is removed by an operator at minute 20.5.
- Total time for 2 packages: 20.5 minutes

Case 2: Fast external system

- The factory opens for the day (time starts).
- Package 1 arrives 1 minute later. It is placed on the conveyor.
- Package 1 takes 1 minute to ride the conveyor belt and is removed by an operator at minute 2.
- Package 2 arrives at minute 2. It is placed on the conveyor.
- Package 2 takes 1 minute to ride the conveyor belt and is removed by an operator at minute 3.
- Total time for 2 packages: 3 minutes

- The factory opens for the day (time starts).
- Package 1 arrives 1 minute later. It is placed on the conveyor.
- Package 1 takes 0.5 minutes to ride the conveyor belt and is removed by an operator at minute 1.5.
- Package 2 arrives at minute 2. It is placed on the conveyor.
- Package 2 takes 0.5 minutes to ride the conveyor belt and is removed by an operator at minute 2.5.
- Total time for 2 packages: 2.5 minutes

In the slow system, the doubling of internal conductivity resulted in a 2.4% reduction in time. In the fast system the doubling of internal conductivity resulted in a 20% reduction in time. The internal change was the same, but it was proportionally more significant in the fast system.

Conclusion

Sensor response time is an important performance parameter that is usually obtained by test measurements. Standard test methods rarely match the heat transfer conditions of actual use and the published results do not always honestly convey the real-life performance of a sensor.

Hopefully this white paper has provided a better understanding of response time and its implications in sensor design. A solid understand of response time is a valuable tool to have for any engineer who designs or specifies temperature sensors.

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